# APPENDIX

# ACHIEVE _ENT TEST

(For diagnose purpose)

**Sub: Mathematics**

Time:1hour 30 min **Class: XII** M.M: 50

**General Instructions**

1. All Questions are compulsory
2. The test paper consists of 25 questions divided into three sections **A, B** and **C. Section A** comprises of **10 questions of one mark** each, **Section B** comprises **10 questions of two marks** each and **Section C comprises 5 questions of four marks each.**
3. **The test paper** is **based** on concepts **Set theory ,Relations and Functions.**

**SECTION A** **(1x10=10)**

**Q1.**Which of the following are sets ( Hint:- Set is a collection of well defined objects)

a) The collection of all months of a year beginning with letter A.
b) The collection of difficult topics in mathematics.
c) The collection of best actors of Bollywood.
d) The collection of all girls in your school.

**Q2.**Describe the given set A={ x:x is a positive integer and -3 ≤x<7} in roster form.

**Q3.**Find the value of a and b if (a-3,b+7)=(2,-5).

**Q4.**Determine the domain and range of the relation R defined by R={(x,x+5):x∈{0,1,2,3,4,5}}.

**Q5.**Give an example of a relation which is symmetric but neither reflexive nor transitive.

**Q6.** Show that the relation R in R defined by R = { (a,b):a≤$b^3$} is neither reflexive nor symmetric nor transitive.

**Q7.** Let R be the relation in the set N given by R= {(a,b): a=b-2, b>6}. Choose the correct answer.

a) (2,4)∈R          c) (3,8)∈R

b) (3,8)∈R          d) (8,7)∈R

**Q8.** Let f:R⟶R be defined as f(x)= $x^2$ then state which type of function f:R ⟶ R is?

**Q9.** What is the necessary condition for gof to be invertible if f:X ⟶Y and g:Y ⟶Z be two invertible functions

**Q10.** If $f:R \longrightarrow R$ be given by $f(x) = (3-x^3)^{1/3}$ then fof(x) is

a) $x^{1/3}$

b) $x^3$

c) x

d) $(3-x^3)$

## SECTION B (2x10=20)

**Q11.** T is a set of triangles and relation $R:T \longrightarrow T$ is given by $R=\{(\Delta_1, \Delta_2) \in T \times T \mid \Delta_1 \cong \Delta_2\}$. Show that R is an equivalence relation.

**Q12.** Show that the relation R defined in the set A of all triangles as $R=\{(T_1, T_2) : T_1$ is similar to $T_2\}$ is equivalence relation. Consider three right angle triangles $T_1$ with sides 3 ,4, 5, $T_2$ with sides 5, 12, 13 and $T_3$ sides 6, 8, 10. Which triangles among $T_1$ ,$T_2$ and $T_3$ are related?

**Q13.** State the difference between one-one function and on-to function.

**Q14.** Are f and g both necessarily onto, if gof is onto ? Justify your answer.

**Q15.** Let $Y= \{n^2 : n \in N\} \subset N$ . Consider $f:N \longrightarrow Y$ as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

**Q16.** Let $f:\{1,3,4\} \longrightarrow \{1,2,5\}$ and $g: \{1,2,5\} \longrightarrow \{1,3\}$ be given by $f=\{(1,2), (3,5), (4,1)\}$ and $g=\{(1,3), (2,3), (5,1)\}$ .Write down (gof).

**Q17.** If A ⊆ B , show that $A x A \subseteq (A \times B) \cap (B \times A)$

**Q18.** Let A ={x,y,z} and B={1,2} . Find the number of relations from A into B.

**Q19.** Let f, g and h be functions from R to R . Show that

(i) (f+g)oh =foh +goh

(ii) (f.g)oh = (foh).(goh)

**Q20.** Let f be defined by f(x)=x-4 and g be defined by

$$g(x) = \begin{cases} \frac{x^2-16}{x+4}, & x \neq -4 \\ k \quad , & x = -4 \end{cases}$$

Find k such that f(x)=g(x) for all x∈R.

**SECTION C** **(4x5=20)**

**Q21.** Find the domain and range of the following

a) f(x)=$\frac{1}{\sqrt{x-5}}$

b) f(x)= $\frac{1}{\sqrt{x+|x|}}$

**Q22.** Find the sum of the identity function and the modulus function.

**Q23.** Let f: N U {0} ⟶ N U {0} be defined by $f(n) = \begin{cases} n+1, if\ n\ is\ even \\ n-1, if\ n\ is\ odd \end{cases}$

show that 'f' is a bijective function.(Hint:- Bijective Functions means one-one on-to functions)

**Q24.** If f(x) =$e^{2x}$ and g(x) = log$\sqrt{x}$ , x>0, find

a) fog
b) f+g
c) gof
d) fg

**Q25.** Let A = {2,3,4,5,6,7,8,9}. Let R be the relation in A defined by {(x,y) : x ∈A , y∈A and x ÷y } find a) R b) domain of R c) range of R d) $R^{-1}$

State whether or not R is a) reflexive b) symmetric c) transtive.

# Intervention through Process approach

## Remedial Measures

**Set:** ⟶ A Collection of well defined objects.

In our daily life, while performing our regular work , we often come across a variety of things that occur in groups.e.g.

1. Army of soldiers
2. Team of cricket platers
3. Group of dancers
4. Pack of playing cards
5. Bunch of beautiful flowers

The words used like Army, Team, Group, Bunch, Pack etc. Convey the idea of certain collections.

For a collection to be a set, that collection must be well defined means elements must be same from any body's point of view.

If any given collection of objects is in such a way that it is possible to tell, without any doubt wether a given object belong to this collection or not, then such a collection of objects is called a well defined collection of objects.

| Not well defined collections | Well defined collections |
|---|---|
| 1. A family of rich persons | • A family of persons having more than ten million rupees. |
| 2. A group of tall students | • A group of students, with height 170cm or more. |

| | |
|---|---|
| 3. A group of numbers | • A group of even natural numbers. |
| 4. A class of intelligent students in class XII | • A class of students, who secured 90% marks in classXII exams. |

**Form to represent a set**



**Roster form**

✧ It means when elements of any set is written in table or list form.

✧**For example** :- Collection of all even numbers less than 10 can be written as

A= {2,4,6,8}

**Set- builder form**

✧ When collection is written in a statement form and person has to build that set.

✧ It mostly used when number of element in any set are very large.

✧**For example**:-A={x:x=3n,n∈N}

**Ordered pair** ⟶ A pair of elements listed in a specific order seperated by comma and enclosing the pair in parenthesis is called an ordered pair.

For example:- (a,b) is an ordered pair with 'a' as the first element and 'b' as the second element.

**NOTE**:- (a,b) $\neq$ (b,a) $\Leftrightarrow$ a=b As we know that graphically the ordered pair (2,3) and (3,2) represents two different points and, hence, they are not equal.

## Cartesian Product of Sets

The set of all ordered pair (a,b) of elements a∈A, b∈B is called the cartesian product of sets A and B, and is denoted by AxB.

For example:- If A={ a,b,c } and B={ p,q } then

i. AxB={(a,p), (a,q), (b,p) (b,q), (c,p), (c,q)}
ii. BxA={(p,a), (q,a), (p,b) (q,b), (p,c), (q,c)}
iii. AxA={(a,a), (a,b), (a,c), (b,a) (b,b), (b,c), (c,a), (c,b), (c,c) }
iv. BxB={(p,p), (p,q), (q,p), (q,q)}

## Relations

The role of relations in our daily life is very important where each relation has its own significance. For example:

I. Relation of mother and son.
II. Relation of wife and husband.
III. Relation of student and teacher.

Similarly, in mathematics also, there is variety of relations, whose knowledge is crucial . Here also each relations has its own meaning and significance . The concept of the term 'relation' in mathematics has been drawn from the meaning of relation in English language, according to which two objects or quantities.

Let us understand this with the help of following examples:

I. *4 is the square of 2* $\Longrightarrow$ *Relation between 2 and 4.*

II. $sinA = \frac{1}{cosecA}$ $\Longrightarrow$ *Relation between sinA and cosecA*

III. *Volume of cube = (edge)3* $\Longrightarrow$ *Relation between volume and edge of a cube.*

**In sets also, we often come across relations such as:**

III. *$x \in A$ i.e., x belongs to A* $\Longrightarrow$ *Relation between x and A.*

IV. *$A \subset B$ i.e., A is the proper subset of B* $\Longrightarrow$ *Relation between A and B*

In all of the above examples, we conclude that every relation involves pairs of objects in a particular order.

Let A be the set of students of class XII of a school and B be the set of students of class XI of the same school. The some of the examples of relations from A to B are

I. { (a,b) ∈ AxB : a is brother of b }
II. { (a,b) ∈ AxB : age of a is greater than age of b }
III. { (a,b) ∈ AxB : total marks obtained by a in the final exams is less than the total marks obtained by b in his final exams }
IV. { (a,b) ∈ AxB : a lives in the same locality as b }

However, abstracting from this , we define mathematically a relation R from A to B as an arbitrary subset of AxB.

If (a,b) ∈ R, we say that a is related to b under the relation R and we write as aRb. In general, (a,b) ∈ R, we do not bother whether there is recognisable connection or link between a and b.

**Relation from set A to set B** ⟶ For finding out relation from set A to set B ,first we have to find out AxB. Then we choose those element from the obtained set AxB which satisfied the given relation.

**For example** :- A={1,2,6} and B={3,7} are two sets and we have to find out Relation of "a is greater then b"⇒ first element is greater then second element.

Then first we find out AxB

AxB={ (1,3), (1,7), (2,3), (2,7), (6,3), (6,7) }

Now seperate those ordered pairs where first element is greater than second element.

Hence relation, R={ (6,3) }

So a relation from A to B is a subset of AxB. and if $(a_1,b_1) \in R$ , we write $a_1 R b_1$.

**Note:-** Let A and B be any two non empty finite sets contaning m and n elements respectively.

Therefore 1). Number of ordered pairs in AxB is mn.

2). Total number of subsets of AxB is $2^{mn}$.

3). Total number of relations from A to B is $2^{mn}$.

because, each relation from A to B is a subset of AxB. Among these $2^{mn}$ relations the void relation∅ and the universal relation AxB are trival relations from A to B.

**Domain of R** ⟶ **$D_R$={a:a∈A, (a,b)∈axB}**

It means domain of R is a collection of all first elements of every ordered pair written in R.

**Range of R** ⟶ $R_R=\{b:b\in B, (a,b)\in axB\}$

It means range of R is a collection of all second elements of every ordered pair written in R.

**Equivalence Relation** ⟶ A relation R on set A is said to be in an equivalence relation if R is reflexive, symmetric and transitive.

**Reflexive**:- If $(a,a)\in R$ for all $a\in A$ or a is related to a for every a

**Symmetric**:- If $(a,b)\in R\Rightarrow(b,a)\in R\ \forall\ a,b\in A$.

**Transitive**:- If $(a,b)\in R$ and $(b,c)\in R \Rightarrow (a,c)\in R\ \forall\ a,b,c\in A$.

or $aRb$ & $bRc \Rightarrow aRc\ \forall\ a,b,c\in A$.

**For example**:-If L is any set of all lines in a plane and R be the relation in L defined as $R=\{ (L_1,L_2):L_1$ is parallel to $L_2 \}$ then it is an equivalence relation because it satisfies all the three given conditions of reflexive , symmetric and transitive.

**For Reflexive**:- If $l_1$ is any line then definetly it is parallel to itself

Since $l_1 \mid\mid l_2 \Rightarrow l_1 R l_1$

$\Rightarrow (l_1,l_2)\in R$

Hence relation R satisfies reflexive property

**For Symmetric**:-If $l_1$ is parallel to $l_2$ then definately $l_2$ is also parallel to $l_1$

thus $l_1 || l_2 \Rightarrow l_2 || l_1$

Hence $(l_1,l_2)$ and $(l_2,l_1)$ both belongs to R.

**For Transitive**:- If $l_1$ is parallel to $l_2$ and $l_2$ is parallel to $l_3$ then $l_1$ is also parallel to $l_3$

since $l_1 || l_2$ and $l_2 || l_3 \Rightarrow l_1 || l_3$

Hence relation R satisfies transitive property.

**Function** ⟶ Function is a special type of relation. In other words, it is a rule that makes new elements out of some given elements.

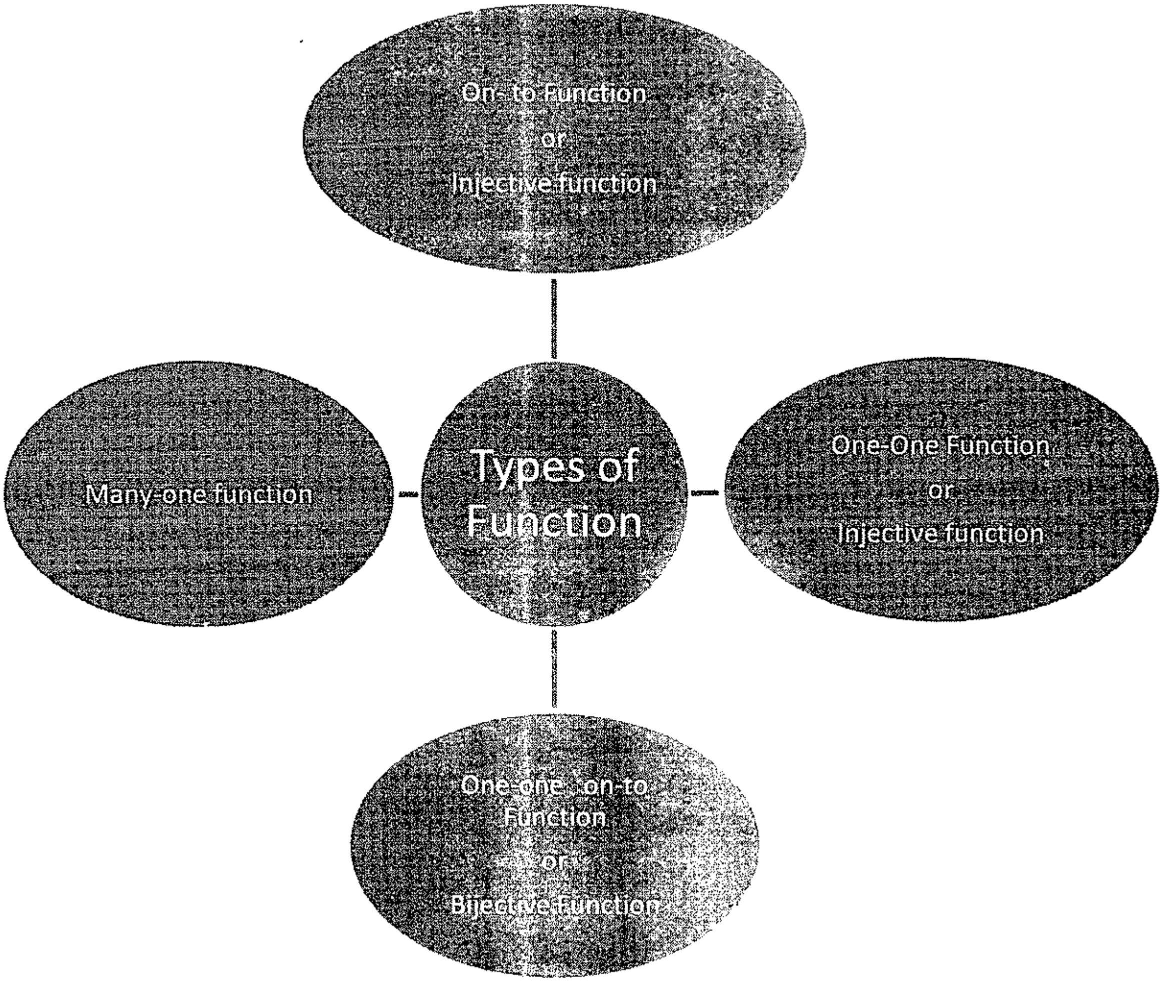
The word 'Function' is derieved from a latin word meaning operation. It is also called by some synonymous words like map and mapping.

A function is a relation from a non- empty set A into, a non empty set B such that:

I. All elements of set A are associated with the elements of set B.
II. An elements of set A is associated with one and only one element of set B. $f:A \longrightarrow B$ such that $\{ x, f(x): x \in A \ \wedge f(x) \in B \}$

A function from A to B is denoted by f and it is written as $f:A \rightarrow B$ or $A \xrightarrow{f} B$

Let A and B are two non empty sets. If there exists a rule 'f' which associates to every element $x \in X$, a unique element $y \in Y$, then f is called a function or mapping from the set X to the set Y. It is represented as $f:X \longrightarrow Y$ and is read as "f is a function from X to Y"




**One -One Function (Injective)** ⟶ A Function $f:X \to Y$ is said to be one-one iff different elements of X have different imajes in Y. i.e., for every $x_1, x_2 \in X$, $f(x_1)=f(x_2) \Rightarrow x_1=x_2$.

**Working rule**:- for checking the function is one-one or not, first we take $f(x_1)=f(x_2)$ then solve it and if we get result as $x_1=x_2$ then we conclude that it is a one-one function.

**For example**:- If f is a function $f:N \to N$ such that $f(x)=x^2$ to check whether it is one-one or not we take $f(x_1)=f(x_2)$

$$\Rightarrow x_1^2=x_2^2$$

$\Rightarrow x1=\sqrt{x_2{}^2}$

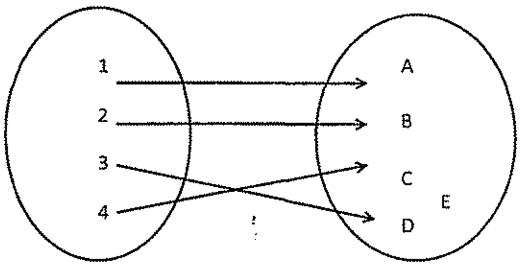
$\Rightarrow x_1=\pm x_2$

[ here we do not consider $-x_2$ because f is from N to N i.e. for natural numbers only. ]

$\Rightarrow x_1=x_2$

Hence f:N⟶N such that f(x) =$x^2$ is a one-one function.

**Graphically**



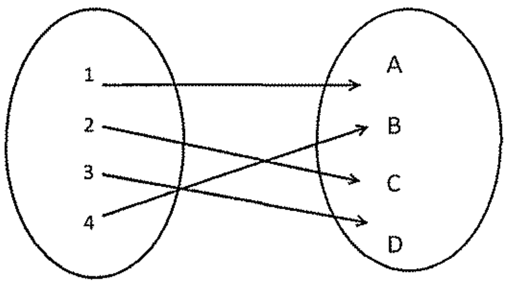
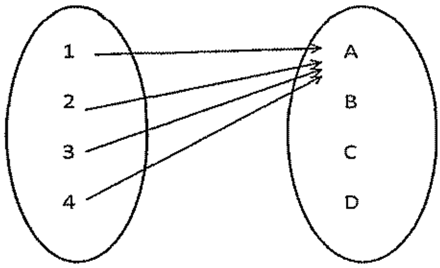




**IMAGE:-** If the element x∈A corresponds to y∈B under the function f, then we say that y is the image of x under f and we write f(x)=y.

**PRE-IMAGE:-** If f(x)=y, then x is the pre-image of y.

**DOMAIN AND CO-DOMAIN**:- The set A is called the domain of function f and the set B is called the co-domain of f.

**Many-One Function**:- If the function is not one-one, then f is calledmany-one.

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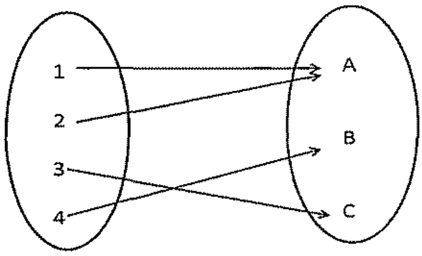
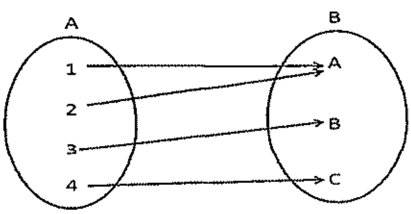
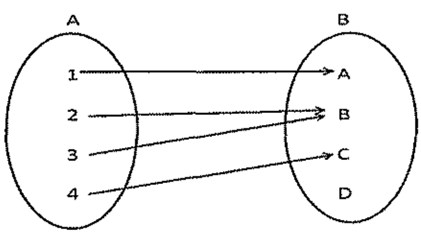



Both are the example of many function

**On-to Function (Surjective)** ⟶ A function f:X ⟶ Y is said to be on-to iff every element in Y is an image of atleast one element in X. i.e., for every y∈Y, there exist an element x∈X such that f(x)=y

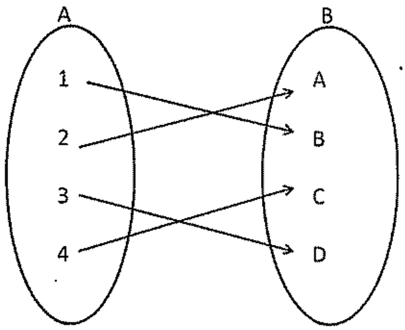



It is an on-to function




It is not an on-to function because in setB one element 'd' is left who is not an image of any element of set A.

**One-One and On-to Function (bijective)** ⟶ If a function is both one-one and on-to then it is said to be one-one on-to function.

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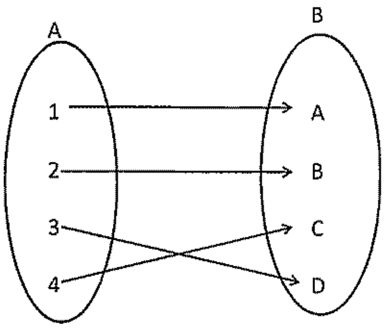


Both are the example of bijective function

**Operations on Functions** ⟶ Let f and g be two real valued functions with domain $D_f$ and $D_g$ respectively. Then the basic operations of addition, subtraction, multiplication and division are defined as

i) Sum: $(f+g)x = f(x) + g(x)$

ii) Difference: $(f-g)x = f(x) - g(x)$

iii) Scalar multiplication : $(cf) = cf(x)$

iv) Product: $(fg)x = f(x)g(x)$

v) Quotient: $(\frac{f}{g})x = \frac{f(x)}{g(x)}$, $g(x) \neq 0$



**Composition of Function** ⟶ Let f and g be two real valued functions with domain $D_f$ and $D_g$ respectively. Let $R_g C D_f$ then the composite of f and g, denoted by fog, is defined as.

$(fog)x = f[g(x)]$ for all $x \in D_g$ In general $fog \neq gof$.

**Invertible Functions** $\longrightarrow$ A function $g:B \to A$ is the invertible function of $f:A \to B$ if and only if $fog = I_B$ and $gof = I_A$.

Thus $f^{-1}(x) = g(x)$.

A Function $f:X \to Y$ is defined to be invertible. If there exists a function $g:Y \to X$ such that $gof = I_X$ and $fog = I_Y$ The function g is called the inverse of f and is denoted by $f^{-1}$.

Thus if f is invertible, then f must be one-one on-to and conversely, if f is one-one and onto, then f must be invertable. This fact significantly helps for proving a function f to be invertible by showing that f is one-one and on-to, specially when the actual inverse of f is not to be determined.

# Item wise Interpretation of the Questions

## Teachers Guide

Q1. For a collection to be a set, that collection must be well defined means elements must be same from any body's point of view.

SET → It is a collection of well defined objects.

**Example1:** Which of the following are sets

a) The collection of all months of a year beginning with letter A.
a) The collection of difficult topics in mathematics.
b) The collection of best actors of Bollywood.
c) The collection of all girls in your school.

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**Solution1**: (a) and (d) both are representing sets , whereas option (b) and (c) are not sets because these collections are completly based on person's own choice.

Q2. **Form of sets**

**Roster form**

✧It means when elements of any set is written in table or list form.

✧**For example** :-Collection of

**Set- builder form**

✧ When collection is written in a statement form and person has to build that set.

✧ It mostly used when number

all even numbers less than 10 can be written as

A= {2,4,6,8}

of element in any set are very large.

✧**For example** :- A={x:x=3n,n∈N}

**Example 2.**Describe the given set A={ x:x is a positive integer and -3 ≤x<7} in roster form.

**Solution 2: A={0,1,2,3,4,5,6}**

**Q3. Ordered pair** ⟶ A pair of elements listed in a specific order seperated by comma and enclosing the pair in parenthesis is called an ordered pair.

**For example**:- (a,b) is an ordered pair with 'a' as the first element and 'b' as the second element.

**NOTE**:- $(a,b) \neq (b,a) \Leftrightarrow a=b$

**Example 3.**Find the value of a and b if (a-3,b+7)=(2,-5).

**Solution3:** A=5 and b=-12

Q4,7,25,19,21.

**Relation from set A to set B** ⟶ For finding out relation from set A to set B ,first we have to find out AxB. Then we choose those element from the obtained set AxB which satisfied the given relation.

**For example** :- A={1,2,6} and B={3,7} are two sets and we have to find out Relation of "a is greater then b"⇒ first element is greater then second element.

Then first we find out AxB

$$AxB=\{ (1,3), (1,7), (2,3), (2,7), (6,3), (6,7) \}$$

Now seperate those ordered pairs where first element is greater than second element.

Hence relation, R={ (6,3) }So a relation from A to B is a subset of AxB. and if $(a_1,b_1)$ЄR , we write $a_1Rb_1$.

**Domain of R** ⟶ **$D_R$={a:aЄA, (a,b)ЄaxB}**

It means domain of R is a collection of all first elements of every ordered pair written in R.

**Range of R** ⟶ **$R_R$={b:bЄB, (a,b)ЄaxB}**

It means range of R is a collection of all second elements of every ordered pair written in R.

**Example 4.**Determine the domain and range of the relation R defined by R={(x,x+5):xЄ{0,1,2,3,4,5}}.

**Solution4:** Domain={0,1,2,3,4,5}

Range={5,6,7,8,9,10}

**Example 5.** Let R be the relation in the set N given by R= {(a,b): a=b-2, b>6}. Choose the correct answer.

a) (2,4)ЄR
b) (3,8)ЄR
c) (3,8)ЄR
d) (8,7)ЄR

**Solution5:** (3,8)ЄR

**Example 6.** Let A = {2,3,4,5,6,7,8,9}. Let R be the relation in A defined by {(x,y) : x∈A , yЄA and x ÷y } find a) R b) domain of R c) range of R

d) $R^{-1}$. State whether or not R is a) reflexive b) symmetric c) transtive.

**Solution 6:** Here, xRy iff x divides y, therefore (i) R= {(2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (3,9), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8), (9,9) }

(ii) Domain of R= {2,3,4,5,6,7,8,9}=A

(iii) Range of R= {2,3,4,5,6,7,8,9}=A

(iv) $R^{-1}$={(y,x): (x,y)∈R}

={(2,2), (4,2), (6,2), (8,2), (3,3), (6,3), (9,3), (4,4), (8,4), (5,5), (6,6), (7,7), (8,8), (9,9) }

**Example 7.** Let f, g and h be functions from R to R . Show that

(i) (f+g)oh =foh +goh

(ii) (f.g)oh = (foh).(goh)

**Solution7**- for all x∈R , we have

(i) ((f+g)oh)(x) = (f+g) (h(x))

= f(h(x))+g(h(x))

= (foh)x+(goh)x

= foh+goh

(ii) ((f.g)oh)(x) =(f.g)(h(x))

= f(h(x)).g(h(x))

=(foh)(x).(goh)(x)

=(foh).(goh)

Thus (f.g)oh =(foh).(goh)

**Example8.** Find the domain and range of the following

$f(x)=\frac{1}{\sqrt{x-5}}$

**Solution8** Domain of f: Clearly,f(x) takes real values, if

$x-5>0 \Longrightarrow x>5 \Longrightarrow x\in(5,\infty)$

Thus Domain of f $=(5,\infty)$

Range of f : for x>5, we have

$x-5>0 \Longrightarrow \sqrt{x-5}>0$

$\Longrightarrow \frac{1}{\sqrt{x-5}}>0$

$\Longrightarrow f(x)>0$

Thus, f(x) takes all real values greater than zero

Hence, Range of f $=(0,\infty)$

## Q5,6,11,12,25

**Equivalence Relation** → A relation R on set A is said to be in an equivalence relation if R is reflexive, symmetric and transitive.

**Reflexive**:- If $(a,a)\in R$ for all $a\in A$ or a is related to a for every a

**Symmetric**:- If $(a,b)\in R \Rightarrow (b,a)\in R\ \forall\ a,b\in A$.

**Transitive**:- If $(a,b)\in R$ and $(b,c)\in R \Rightarrow (a,c)\in R\ \forall\ a,b,c\in A$.

or aRb & bRc $\Rightarrow$ aRc $\forall\ a,b,c\in A$.

**For example**:-If L is any set of all lines in a plane and R be the relation in L defined as $R=\{(L_1,L_2):L_1$ is parallel to $L_2\}$ then it is an equivalence relation because it satisfies all the three given conditions of reflexive , symmetric and transitive.

**For Reflexive**:- If $l_1$ is any line then definetly it is parallel to itself

Since $l_1 \,||\, l_2 \Rightarrow l_1 R l_1$

$\Rightarrow (l_1, l_2) \in R$

Hence relation R satisfies reflexive property

**For Symmetric**:-If $l_1$ is parallel to $l_2$ then definately $l_2$ is also parallel to $l_1$

thus $l_1 || l_2 \Rightarrow l_2 || l_1$

Hence $(l_1, l_2)$ and $(l_2, l_1)$ both belongs to R.

**For Transitive**:- If $l_1$ is parallel to $l_2$ and $l_2$ is parallel to $l_3$ then $l_1$ is also parallel to $l_3$

since $l_1 || l_2$ and $l_2 || l_3 \Rightarrow l_1 || l_3$

Hence relation R satisfies transitive property.

**Q5.**Give an example of a relation which is symmetric but neither reflexive nor transitive.

Let A={1,2,3} and R= {(2,3),(3,2)}

Then the relation R is symmetric because $(2,3) \in R$ and $(3,2) \in R$,But Since $(1,1) \notin R$, therefore R is not reflexive

Also $(2,3) \in R$, $(3,2) \in R$ but $(2,2) \notin R$, therefore R is not transitive.

Q6.Show that the relation R in R defined by R = { (a,b):$a \le b^3$} is neither reflexive nor symmetric nor transitive.

Reflexivity--- As $a \le a^3$ is not true for all a$\in$R,

(e.g for $a=\frac{1}{2}$ , $a>a^3$ ),

therefore, R is not reflexive.

Symmetry--- For a,b $\in$ R, $a \le b^3$ need not imply $b \le a^3$,

therefore R is not symmetric.

Transitive--- For a,b,c $\in$R, $a \le b^3$and $b \le c^3$need not imply

$a \le c^3$ , therefore, aRb and bRc$\not\Leftrightarrow$ aRc.

therefore it is not transitive.

## Q8,13,14,22,23

**Function** $\longrightarrow$ Let A and B are two non empty sets. If there exists a rule 'f' which associates to every element x$\in$X, a unique element y$\in$Y, then f is called a function or mapping from the set X to the set Y. It is represented as f:X $\rightarrow$Y and is read as "f is a function from X to Y"

**One -One Function (Injective)** $\longrightarrow$ A Function f:X $\rightarrow$ Y is said to be one-one iff different elements of X have different images in Y. i.e., for every $x_1,x_2 \in X$, $f(x_1)=f(x_2) \Rightarrow x_1=x_2$.

**Working rule**:- for checking the function is one-one or not, first we take $f(x_1)=f(x_2)$ then solve it and if we get result as $x_1=x_2$ then we conclude that it is a one-one function.

**For example**:- If f is a function $f:N \rightarrow N$ such that $f(x)=x^2$ to check whether it is one-one or not we take $f(x_1)=f(x_2)$

$$\Rightarrow x_1^2=x_2^2$$

$$\Rightarrow x1=\sqrt{x_2^2}$$

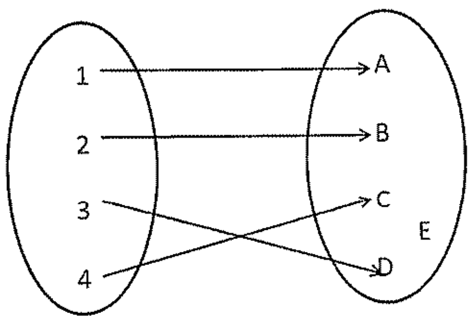
$$\Rightarrow x_1=\pm x_2$$

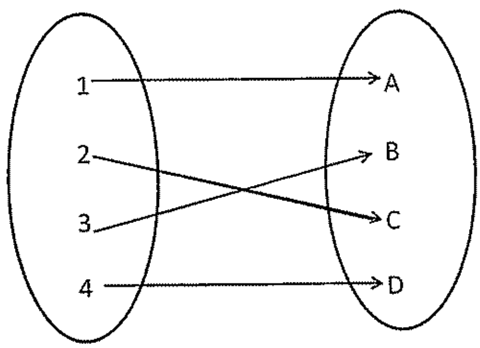
[ here we do not consider $-x_2$ because f is from N to N i.e. for natural numbers only. ]

$$\Rightarrow x_1=x_2$$

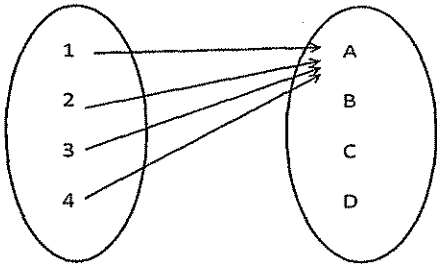
Hence $f:N \rightarrow N$ such that $f(x)=x^2$ is a one-one function.

**Graphically**



**Many-One Function**:- If the function is not one-one, then f is calledmany-one.

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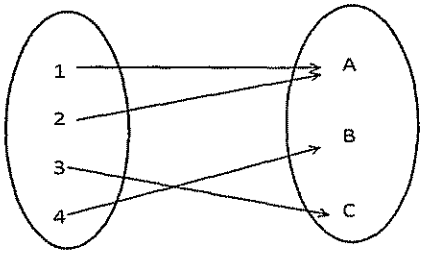
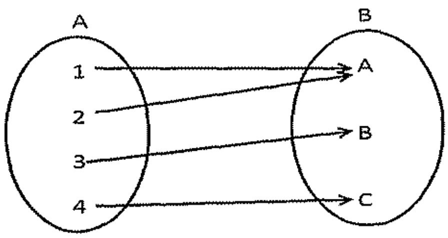
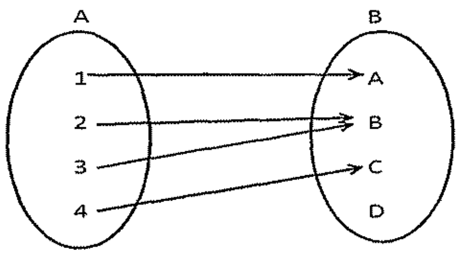



Both are the example of many function

**On-to Function (Surjective)** ⟶ A function f:X ⟶ Y is said to be on-to iff every element in Y is an image of atleast one element in X. i.e., for every y∈Y, there exist an element x∈X such that f(x)=y

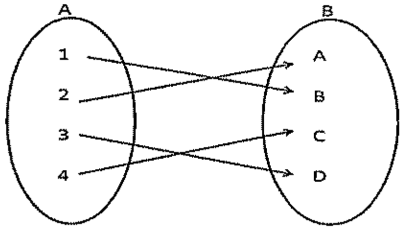



It is an on-to function




It is not an on-to function because in setB oneelement'd' is left who is notan image of any element of set A.

**One-One and On-to Function (bijective)** ⟶ If a function is both one-one and on-to then it is said to be one-one on-to function.

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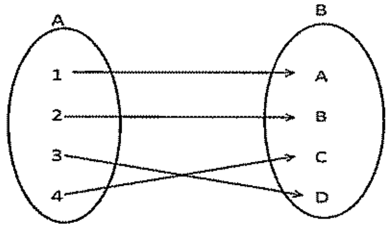


Both are the example of bijective function

Q9,10,15,16,24

**Operations on Functions** ⟶ Let f and g be two real valued functions with domain $D_f$ and $D_g$ respectively. Then the basic operations of addition, subtraction, multiplication and division are defined as

i) Sum: $(f+g)x = f(x) + g(x)$

ii) Difference: $(f-g)x = f(x) - g(x)$

iii) Scalar multiplication : $(cf) = cf(x)$

iv) Product: $(fg)x = f(x)g(x)$

v) Quotient: $(\frac{f}{g})x = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

**Composition of Function** ⟶ Let f and g be two real valued functions with domain $D_f$ and $D_g$ respectively. Let $R_g C D_f$ then the composite of f and g, denoted by fog, is defined as.

$(fog)x = f[g(x)]$ for all $x \in D_g$ In general $fog \neq gof$.

**Invertible Functions** ⟶ A function $g:B \to A$ is the invertible function of $f:A \to B$ if and only if $fog = I_B$ and $gof = I_A$ .

Thus $f^{-1}(x) = g(x)$.

A Function $f:X \rightarrow Y$ is defined to be invertible. If there exists a function $g:Y \rightarrow X$ such that $gof=I_X$ and $fog = I_Y$ The function g is called the inverse of f and is denoted by $f^{-1}$.

Thus if f is invertible , then f must be one-one on-to and conversely, if f is one-one and onto, then f must be invertable. This fact significantly helps for proving a function f to be invertible by showing that f is one-one and on-to, specially when the actual inverse of f is not to be determined.